

COMMON FIXED-POINT THEOREMS FOR GENERALIZED QUANTUM SEARCH OPERATORS IN COMPLETE FUZZY METRIC SPACES

T. Rakesh Singh^{1*}, Eedi Hemalatha²

¹ Aurora's Scientific and Technological Institute, Aushapur (V), Ghatkesar (M), Medchal District, Hyderabad-501301, Telangana, India. (rakeshsinghthakur16@gmail.com)

² University College of Engineering, Science & Technology Hyderabad (UCEST), Kukatpally, Hyderabad-500085, Telangana, India. (hemamorarjee@jntuh.ac.in)

Corresponding Author Email: rakeshsinghthakur16@gmail.com

ABSTRACT

Quantum-search-inspired processes can be represented as iterative transformations in which a computational state is successively modified toward a limiting state. To provide a mathematical framework for analyzing such processes under abstract uncertainty, this paper develops a common fixed-point approach for two generalized search operators defined on a complete fuzzy metric space. A generalized Wardowski-type F -contractive condition is formulated in which the fuzzy nearness between the images of the two operators is controlled by the original-state nearness together with operator-de-pendent fuzzy displacements. The elements of the fuzzy metric space are interpreted as abstract computational states, while the two self-mappings represent successive search and refinement transformations. The complete search-refinement step is described by the composite transformation $T = Q_2 \circ Q_1$. The proposed framework does not claim a direct physical simulation of a quantum device or a specific quantum noise channel; rather, fuzzy nearness is employed as an abstract representation of uncertainty in an iterative computational process. A numerical example with multiple initial states is presented to illustrate the convergence behaviour of the proposed fixed-point framework. The results demonstrate how common fixed-point theory in complete fuzzy metric spaces can be used to study convergence of generalized search-refinement processes under abstract uncertainty.

Keywords: Quantum search algorithms, Fuzzy metric spaces, Wardowski F-contractions, Quantum computing, Convergence Analysis, Stability Analysis, Quantum Optimization, Common Fixed Point.

1. INTRODUCTION

Quantum computing has emerged as an important area of computational research by exploiting quantum-mechanical principles such as superposition, entanglement, and interference [1]. These principles provide computational mechanisms that differ fundamentally from those of classical computing [2]. Among quantum algorithms, quantum search procedures are particularly important because they provide efficient approaches for locating desired states in large search spaces [3]. Grover's algorithm is a well-known example that provides a quadratic query-complexity improvement for unstructured search under the standard oracle model [4]. The iterative structure of search procedures also motivates mathematical investigations of state evolution, convergence, and stability [5].

An iterative search-refinement process can be represented abstractly as a sequence of transformations applied successively to a computational state [6]. If the state at iteration n is denoted by x_n , a general two-stage process can be represented as

$$x_{n+1} = Q_2(Q_1(x_n)) \quad (1)$$

where Q_1 represents a search transformation and Q_2 represents a subsequent refinement or state-update transformation. Introducing the composite operator

$$T = Q_2 \circ Q_1 \quad (2)$$

The complete search-refinement step can be written as

$$x_{n+1} = T(x_n) \quad (3)$$

This representation provides a natural mathematical setting for studying the limiting behavior of repeated search-refinement transformations by means of fixed point theory.

Banach's contraction principle forms one of its fundamental results and has subsequently been extended through numerous generalized contractive conditions [7] [8]. Among these developments, Wardowski-type F-contractions provide a general framework in which the conventional linear contraction requirement is replaced by a condition involving an auxiliary function Φ . Common fixed point theory further extends this framework to pairs or families of interacting mappings and is therefore relevant to iterative processes involving multiple successive transformations [9].

Fuzzy metric spaces provide a mathematical framework for describing degrees of nearness rather than relying exclusively on exact distance values [10]. In an iterative computational

setting, fuzzy nearness can be used to describe the gradual similarity between an evolving state and a limiting state [11]. This motivates the investigation of fixed-point properties of generalized search-refinement transformations in complete fuzzy metric spaces [12].

The present study addresses the specific mathematical problem of formulating a two-stage search-refinement process within a fuzzy fixed-point framework [13] [14]. In particular, a fixed-point result formulated only for a single transformation does not explicitly represent the successive action of two operators when one complete iteration is described by their composition [15]. The present work therefore considers a pair of generalized quantum-search-inspired operators and explicitly represents their complete action through the composite transformation

$$T = Q_2 \circ Q_1 \quad (4)$$

The objective of this study is to establish a common fixed-point result for two continuous self-mappings

$$Q_1, Q_2 \rightarrow X \quad (5)$$

on a complete fuzzy metric space $(X, M, *)$. The mappings are interpreted as successive transformations of an abstract computational state, while their complete action is represented by

$$T = Q_2 \circ Q_1 \quad (6)$$

Under a generalized Wardowski-type F-contractive condition, sufficient conditions are established for the existence and uniqueness of the common fixed point associated with the proposed operator pair and for the convergence of the corresponding iterative process [16]. The resulting iteration is then illustrated using a numerical density-operator model in which the selected transformations generate an explicitly convergent sequence [17].

The contribution of the present work is the formulation and analysis of a two-stage quantum-search-inspired operator framework within a complete fuzzy metric setting [18]. Rather than claiming a physical implementation of a quantum search algorithm, the study provides an abstract mathematical representation in which successive search and refinement transformations are analyzed through common fixed-point theory [19]. The numerical construction further illustrates the connection between the theoretical fixed-point result, the generated state sequence, and the corresponding fuzzy similarity [20].

The principal contributions of the present study are summarized as follows

1. A two-stage generalized quantum-search operator framework is formulated on a complete fuzzy metric space, with one complete search-refinement step represented by the composite transformation $T = Q_2 \circ Q_1$ (6)
2. A generalized Wardowski-type F-contractive condition is formulated for the proposed operator pair, providing

sufficient conditions for the convergence analysis of the associated composite transformation.

3. The theoretical result is connected explicitly with the iterative search-refinement process through

$$x_{n+1} = Q_2(Q_1(x_n)) \quad (7)$$

4. A numerical density-operator model is constructed to illustrate the theoretical framework. In particular, the selected operators generate the composite transformation

$$T(\rho) = 0.3\rho + 0.7\rho^* \quad (8)$$

5. which produces geometric convergence toward the limiting state ρ^* .
6. The convergence behavior is interpreted in terms of the fuzzy metric, illustrating how increasing fuzzy similarity can characterize the approach of the iterated computational state toward its fixed point.

2. PRELIMINARIES

In this section, we recall the basic concepts of fuzzy metric spaces, convergence, completeness, and Wardowski-type F-functions that are required for the subsequent analysis. Throughout the paper, $(X, M, *)$ denotes a fuzzy metric space in the sense of George and Veeramani, where X is a non-empty set, M is the fuzzy metric, and $*$ is a continuous triangular norm.

Definition 2.1.

Let $(X, M, *)$ be a fuzzy metric space and let $Q_1, Q_2: X \rightarrow X$ be self-mappings. A point $x^* \in X$ is called a common fixed point of Q_1 and Q_2 if $Q_1 x^* = Q_2 x^* = x^*$.

For the iterative process considered in this paper, one complete search-refinement step is represented by the composite mapping

$$T = Q_2 \circ Q_1. \quad (7)$$

Thus, for an initial state $x_0 \in X$, the associated sequence is generated by $x_{n+1} = T(x_n) = Q_2(Q_1(x_n))$, $n = 0, 1, 2, \dots$

A fixed point of the composite transformation satisfies

$$T(x^*) = x^*, \text{ that is, } Q_2(Q_1(x^*)) = x^*. \quad (8)$$

This composite fixed-point formulation is the one used in the subsequent convergence analysis and numerical application.

Remark 2.2.

The distinction between a common fixed point of Q_1 and Q_2 and a fixed point of the composite transformation $T = Q_2 \circ Q_1$ is important. In general,

$$Q_2(Q_1(x^*)) = x^* \quad (9)$$

does not by itself imply

$$Q_1(x^*) = Q_2(x^*) = x^* \quad (10)$$

Accordingly, the main iterative analysis in this paper is formulated in terms of the fixed point of the composite search-refinement transformation T .

3. GENERALIZED QUANTUM SEARCH OPERATORS

The convergence of iterative quantum search algorithms is governed by the repeated application of quantum operators that transform an initial quantum state into a desired target state [21]. Motivated by this observation, we introduce a generalized mathematical model for quantum search processes in complete fuzzy metric spaces. The proposed framework does not depend on a particular quantum search algorithm, such as Grover's algorithm, but instead considers a broad class of iterative quantum search operators whose convergence can be investigated through common fixed-point theory.

Let $(X, M, *)$ be a complete fuzzy metric space. The elements of X represent generalized quantum states, while the mappings defined on X describe successive quantum search transformations.

Definition 3.1.

(Generalized Quantum Search Operator Pair).

Let $(X, M, *)$ be a complete fuzzy metric space and let $Q_1, Q_2: X \rightarrow X$ be two self-mappings. The pair (Q_1, Q_2) is called a generalized quantum search operator pair if Q_1 and Q_2 represent two successive transformations of an abstract computational state, where Q_1 denotes a search transformation and Q_2 denotes a refinement or stabilization transformation [22].

The iterative process associated with the pair is defined by $x_{n+1} = Q_2(Q_1(x_n))$, $n = 0, 1, 2, \dots$, for an initial state $x_0 \in X$.

A point $x^* \in X$ is called a stable common state of the operator pair if $Q_1(x^*) = Q_2(x^*) = x^*$.

The existence and uniqueness of such a stable common state are not assumed in this definition and will be established under suitable contractive conditions [23].

In the present work, X is regarded as an abstract state or parameter space used to represent the evolving states of a generalized search process. Thus, the elements of X are not assumed to be physical state vectors or density operators unless an additional embedding into a quantum state space is specified. Similarly, Q_1 and Q_2 are treated as abstract self-mappings representing successive search and refinement transformations [24]. This abstraction allows the convergence properties of the process to be studied using fuzzy metric and fixed-point theory without imposing a particular physical implementation [25].

Definition 3.2[Fuzzy Distance]

Let $(X, M, *)$ be a fuzzy metric space. For $x, y \in X$ and $t > 0$, define the fuzzy distance deficit by

$$d_t(x, y) = 1 - M(x, y, t).$$

Then

$$0 \leq d_t(x, y) < 1, \text{ and } d_t(x, y) = 0 \Leftrightarrow x = y.$$

Moreover,

$$M(x_n, x, t) \rightarrow 1 \text{ if and only if } d_t(x_n, x) \rightarrow 0.$$

Thus, $d_t(x, y)$ provides a convenient distance-like measure of the failure of fuzzy nearness between two points.

4. MAIN THEOREM

Let $(X, M, *)$ be a complete fuzzy metric space. In this section, we establish a common fixed-point result for the generalized quantum search operator pair (Q_1, Q_2) . To maintain consistency between the mathematical analysis and the iterative search process, we consider the composite transformation $T = Q_2 \circ Q_1$. The composite transformation represents one complete search-refinement step.

The two-operator fuzzy F-contractive condition introduced in Definition 3.2 describes the interaction between the search and refinement mappings. For the convergence proof, we additionally require the composite transformation T to satisfy a Wardowski-type F-contractive condition. This additional assumption provides a direct contraction mechanism for the actual iterative process and avoids inferring the Cauchy property solely from the monotonicity of successive fuzzy-nearness values.

Theorem 4.1. [Common Fixed-Point Theorem]

Let $(X, M, *)$ be a complete fuzzy metric space, and let $Q_1, Q_2: X \rightarrow X$ be continuous self-mappings. Define the composite transformation

$$T = Q_2 \circ Q_1.$$

For $x, y \in X$ and $t > 0$, let $d_t(x, y) = 1 - M(x, y, t)$. Suppose that there exist a Wardowski F-function

$$F: (0, 1] \rightarrow \mathbf{R} \text{ and a constant } \tau > 0 \text{ such that} \\ \tau + F(d_t(Tx, Ty)) \leq F(d_t(x, y)) \text{ ---- (4.1)} \\ \text{for all } x, y \in X, x \neq y, \text{ and } t > 0 \text{ for which } d_t(x, y) > 0.$$

Assume further that the fixed-point sets of the composite transformation and the two-operator pair satisfy

$$\text{Fix}(T) = \text{Fix}(Q_1) \cap \text{Fix}(Q_2). \text{ ----- (4.2)}$$

For an arbitrary initial point $x_0 \in X$, define

$$x_{n+1} = T(x_n) = Q_2(Q_1(x_n)), \quad n = 0, 1, 2, \dots \text{--- (4.3)}$$

Assume that the sequence x_n generated by (4.3) satisfies the standard fuzzy Cauchy criterion.

Applying the contractive condition (4.1) to $x = x_n$ and $y = x_{n-1}$ gives

$$\tau + F(d_t(x_{n+1}, x_n)) \leq F(d_t(x_n, x_{n-1})).$$

Therefore,

$$F(d_t(x_{n+1}, x_n)) \leq F(d_t(x_n, x_{n-1})) - \tau.$$

By induction,

$$F(d_t(x_{n+1}, x_n)) \leq F(d_t(x_1, x_0)) - n\tau. \text{ ---- (4.4)}$$

Since $\tau > 0$,

$$F(d_t(x_{n+1}, x_n)) \rightarrow -\infty \text{ as } n \rightarrow \infty$$

By the defining property of the Wardowski F-function,

$\lim_{r \rightarrow 0^+} F(r) = -\infty$
and hence

$$d_t(x_{n+1}, x_n) \rightarrow 0.$$

Since

$$d_t(x, y) = 1 - M(x, y, t), \text{ we obtain}$$

$$M(x_{n+1}, x_n, t) \rightarrow 1, \text{ as } n \rightarrow \infty \text{ ---- (4.5)}$$

By assumption, the Picard sequence $\{x_n\}$ satisfies the standard fuzzy Cauchy criterion. Hence, for every $\varepsilon \in (0, 1)$ and $t > 0$, there exists $N \in \mathbb{N}$ such that

$$M(x_m, x_n, t) > 1 - \varepsilon, \text{ for all } m, n \geq N.$$

Thus $\{x_n\}$ is a Cauchy sequence in $(X, M, *)$.

Since $(X, M, *)$ is complete, there exists $x^* \in X$ such that

$$x_n \rightarrow x^*. \text{ ---- (4.6)}$$

Because Q_1 and Q_2 are continuous, the composite mapping

$$T = Q_2 \circ Q_1 \text{ is continuous. Therefore,}$$

$$x_{n+1} = T(x_n) \rightarrow T(x^*).$$

On the other hand, since $x_n \rightarrow x^*$, the shifted sequence

$\{x_{n+1}\}$ also converges to x^* . By uniqueness of the fuzzy Metric limit,

$$T(x^*) = x^* \text{ ---- (4.7)}$$

Thus,

$$x^* \in \text{Fix}(T).$$

By assumption (4.2),

$$\text{Fix}(T) = \text{Fix}(Q_1) \cap \text{Fix}(Q_2),$$

and therefore

$$Q_1(x^*) = x^*, Q_2(x^*) = x^* \text{ ---- (4.8)}$$

Hence x^* is a common fixed point of Q_1 and Q_2 .

To prove uniqueness, suppose that z^* is another common fixed point of Q_1 and Q_2 . Then

$$T(z^*) = Q_2(Q_1(z^*)) = z^*.$$

Suppose that $x^* \neq z^*$. Then

$d_t(x^*, z^*) > 0$ for every $t > 0$. Applying (4.1) with $x = x^*$ and $y = z^*$ gives

$$\tau + F(d_t(Tx^*, Tz^*)) \leq F(d_t(x^*, z^*)).$$

Since $Tx^* = x^*$ and $Tz^* = z^*$,

$$\tau + F(d_t(x^*, z^*)) \leq F(d_t(x^*, z^*)),$$

which is impossible because $\tau > 0$.

Therefore,

$$x^* = z^*.$$

Hence T has a unique fixed point, and this point is also the unique common fixed point of Q_1 and Q_2 .

Corollary 4.2. [Quantum Search Stability]

Under the assumptions of Theorem 4.1, let

$$x_{n+1} = Q_2(Q_1(x_n)), \quad n = 0, 1, 2, \dots \text{ ---- (4.9)}$$

Then the iterative quantum-search process converges to the unique common fixed point x^* satisfying

$$Q_1(x^*) = Q_2(x^*) = x^*.$$

Hence, the proposed fuzzy metric framework provides a sufficient mathematical condition for the convergence of the generalized quantum-search state transformation under abstract fuzzy uncertainty. The limiting state is independent

of the initial state whenever the hypotheses of Theorem 4.1 hold.

5. APPLICATION AND INTERPRETATION IN QUANTUM SEARCH

The numerical example presented in Section 5 illustrates the proposed generalized quantum search operator framework on the abstract computational state space

$$X = [0, 1].$$

The Numerical convergence of the search-refinement iteration is presented in table 1.

The two successive transformations are defined by

$$Q_1(x) = 0.5x + 0.3 \text{ and}$$

$$Q_2(x) = 0.6x + 0.24.$$

Consequently, the composite search-refinement transformation is

$$T(x) = Q_2(Q_1(x)) = 0.3x + 0.42.$$

The fixed point of the composite transformation is obtained from

$$x^* = T(x^*) = 0.3x^* + 0.42, \text{ which gives } x^* = 0.6.$$

Moreover, $Q_1(0.6) = 0.6$ and $Q_2(0.6) = 0.6$.

Table 1. Numerical convergence of the search-refinement iteration.

n	x_n	$x_n - x^*$	$M(x_n, x^*)$
0	0.200000	0.400000	0.670320
1	0.480000	0.120000	0.886920
2	0.564000	0.036000	0.964641
3	0.589200	0.010800	0.989258
4	0.596760	0.003240	0.996765
5	0.599028	0.000972	0.999029
6	0.599708	0.000292	0.999708
7	0.599913	0.000087	0.999913
8	0.599974	0.000026	0.999974
9	0.599992	0.000008	0.999992
10	0.599998	0.000002	0.999998

Hence,

$$Q_1(x^*) = Q_2(x^*) = x^* = 0.6,$$

showing that $x^* = 0.6$ is a common fixed point of the two transformations, consistent with the fixed-point framework established in Section 4. The Numerical convergences of the search-refinement iteration are given in Table 1.

The corresponding search-refinement iteration is

$$x_{n+1} = Q_2(Q_1(x_n)),$$

$$\text{or, equivalently, } x_{n+1} = 0.3x_n + 0.42.$$

Starting from the initial state $x_0 = 0.2$, the first few iterates are $x_1 = 0.48$, $x_2 = 0.564$, $x_3 = 0.5892$, and the sequence continues toward $x^* = 0.6$. This numerical convergence shown in Figure 1 agrees with the fixed-point result established in Section 4 and the numerical analysis presented in Section 5.

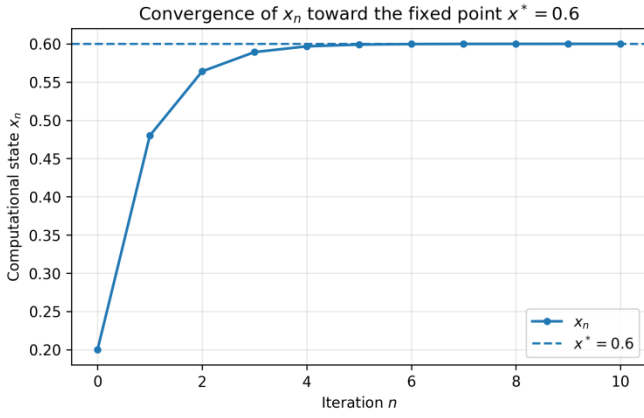


Figure 1. Convergence of the computational state x_n toward the fixed point $x^* = 0.6$.

Within the proposed framework, Q_1 represents the first stage of the abstract search transformation, while Q_2 represents the subsequent refinement or stabilization transformation. Their composition

$$T = Q_2 \circ Q_1$$

therefore represents one complete search-refinement cycle.

Repeated application of this composite transformation produces the sequence

$$x_{n+1} = T(x_n),$$

which converges to the common fixed-point $x^* = 0.6$ for the numerical example considered here.

The fuzzy metric used in Section 5 provides a measure of the similarity between two computational states. In particular, with $M(x, y, t) = e^{-|x-y|}$, the convergence $x_n \rightarrow x^*$

Implies $M(x_n, x^*, t) \rightarrow 1$.

Thus, the fuzzy similarity between the iterated computational state and the limiting state approaches its maximum value as the iteration proceeds as depicted in Figure 2. It is important to emphasize that the state space $X = [0, 1]$ used in this example is an abstract computational state space.

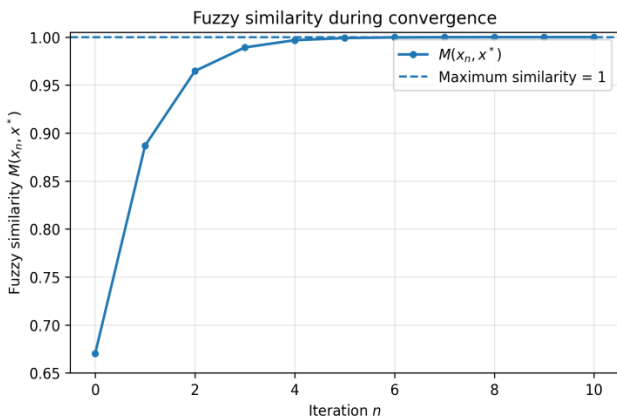


Figure 2. Fuzzy similarity $M(x_n, x^*)$ during the search-refinement iteration.

In particular, the transformations Q_1 and Q_2 are abstract mappings representing successive search and refinement stages the same is shown in Figure 2.

Therefore, the numerical example demonstrates how the common fixed-point framework can be used to describe convergence and stability of an abstract quantum-search-inspired iterative process. The common fixed point

$$x^* = 0.6$$

represents the stable computational state reached through repeated application of the composite transformation

$$T = Q_2 \circ Q_1$$

6. CONCLUSION AND FUTURE SCOPE

In this paper, a common fixed-point framework for generalized quantum search operators has been developed in complete fuzzy metric spaces. Two successive transformations, Q_1 and Q_2 , were considered, and their composition

$$T = Q_2 \circ Q_1$$

was used to represent one complete search-refinement step. A Wardowski-type F-contractive condition was imposed on the composite transformation, and a common fixed-point result was established under the stated completeness, continuity, and fixed-point assumptions.

The proposed framework provides a mathematical description of convergence and stability for an abstract quantum-search-inspired iterative process. The numerical example in Section 5, based on the computational state space $X = [0, 1]$, demonstrates the iterative behavior of the transformations

$$Q_1(x) = 0.5x + 0.3 \text{ and } Q_2(x) = 0.6x + 0.24.$$

Their composite transformation is

$$T(x) = 0.3x + 0.42,$$

which has the common fixed-point $x^* = 0.6$. The numerical iterations confirm convergence toward this stable computational state, while the fuzzy metric formulation describes the corresponding increase in similarity to the limiting state.

The application discussed in Section 6 interprets Q_1 and Q_2 as abstract successive search and refinement transformations. The proposed formulation should be viewed as a mathematical framework rather than as a physical implementation of a specific quantum search algorithm. Its main purpose is to connect fixed-point theory, fuzzy convergence, and an abstract search-refinement process within a unified mathematical setting.

6.1 Future Scope

Several directions can be considered for future research. The present framework may be extended to more general fuzzy metric structures, alternative contractive conditions, and multi-stage search-refinement processes involving more than two transformations. Further work may also investigate applications to specific quantum-inspired optimization, ranking, and search models by defining appropriate computational state spaces and operators. Another possible direction is to study computational implementations of the proposed fixed-point framework and compare the theoretical convergence behavior with numerical simulations for larger

state spaces. Extensions to other generalized metric spaces and hybrid fixed-point models may also provide additional mathematical and computational applications.

ACKNOWLEDGMENT

The authors sincerely acknowledge the valuable guidance, support, and constructive suggestions received during the course of this research. The authors also thank the reviewers for their insightful comments, which helped improve the quality of this manuscript.

REFERENCES

- [1]. S. Banach, “Sur les opérations dans les ensembles abstraits et leur application aux équations intégrales”, in *Fundamenta Mathematicae*, 3, 133-181, 1992.
- [2]. D. Wardowski, “Fixed points of a new type of contractive mappings in complete metric spaces”, *Fixed Point Theory and Applications*, Article 94, 2012
- [3]. D. Wardowski, “Fuzzy contractive mappings and fixed points in fuzzy metric spaces,” *Fuzzy Sets and Systems*, 222, 108-114, 2013.
- [4]. N. A. Secelean, “Iterated function systems consisting of F-contractions”, in *Fixed Point Theory and Applications*, Article 277, 2013
- [5]. S. Chandok, H. Huang and S. Radenović, “Generalized F-Suzuki contractions in b-metric spaces”, in *Sahand Communications in Mathematical Analysis*, 5, 23 - 36, 2018.
- [6]. L. A. Zadeh, “Fuzzy sets”, in *Information and Control*, 8, 338 – 353, 1965.
- [7]. A. George and P. Veeramani, “On some results in fuzzy metric spaces”, in *Fuzzy Sets and Systems*, 64, 395 - - 399, 1994.
- [8]. M. Grabiec, “Fixed points in fuzzy metric spaces”, in *Fuzzy Sets and Systems*, 27, 385 – 389, 1988.
- [9]. V. Gregori and A. Sapena, “On fixed-point theorems in fuzzy metric spaces”, in *Fuzzy Sets and Systems*, 125, 245 - 252, 2002.
- [10]. D. Mihet, “Fuzzy ϕ -contractive mappings in non-Archimedean fuzzy metric spaces”, in *Fuzzy Sets and Systems*, 159, 739 - 744, 2008.
- [11]. V. Gregori, J. J. Miñana and D. Miravet, “Contractive sequences in fuzzy metric spaces”, in *Fuzzy Sets and Systems*, 379, 125 - 133, 2020.
- [12]. A. Moussaoui, N. Hussain, S. Melliani and others, “Fixed point results via extended FZ-simulation functions in fuzzy metric spaces”, in *Journal of Inequalities and Applications*, Article 85, 2022.
- [13]. A. Moussaoui and S. Radenović, “Fuzzy metric spaces: A survey on fixed point results, contraction principles and simulation functions”, in *Fixed Point Theory and Algorithms for Sciences and Engineering*, 2025.
- [14]. M. A. Nielsen and I. L. Chuang, “Quantum Computation and Quantum Information”, in Cambridge University Press, Cambridge, 10th Anniversary Edition, 2010.
- [15]. L. K. Grover, “A fast quantum mechanical algorithm for database search”, in *Proceedings of the Twenty-Eighth Annual ACM Symposium on Theory of Computing (STOC)*, pp. 212 – 219, 1996.
- [16]. L. K. Grover, “Quantum mechanics helps in searching for a needle in a haystack”, in *Physical Review Letters*, 79, 325 - 328, 1997.
- [17]. G. Brassard, P. Høyer, M. Mosca and A. Tapp, “Quantum amplitude amplification and estimation”, in *Contemporary Mathematics*, 305, 53 – 74, 2002.
- [18]. A. Ambainis, “Quantum walk algorithm for element distinctness”, in *SIAM Journal on Computing*, 37, 210 - 239, 2007.
- [19]. E. Farhi, J. Goldstone and S. Gutmann, “A quantum algorithm for the Hamiltonian NAND tree”, in *Theory of Computing*, 4, 169 – 190, 2008.
- [20]. J. Preskill, “Quantum computing in the NISQ era and beyond”, in *Quantum*, 2, Article 79, 2018.
- [21]. M. Cerezo, A. Arrasmith, R. Babbush and others, “Variational quantum algorithms”, in *Nature Reviews Physics*, 3, 625 – 644, 2021.
- [22]. J. Biamonte, P. Wittek, N. Pancotti, P. Rebentrost, N. Wiebe and S. Lloyd, “Quantum machine learning”, in *Nature*, 549, 195–202, 2017.
- [23]. B. Schweizer and A. Sklar, “Statistical metric spaces”, *Pacific Journal of Mathematics*, 10, 313 – 334, 1960.
- [24]. V. M. Sehgal and A. T. Bharucha-Reid, “Fixed points of contraction mappings on probabilistic metric spaces”, in *Mathematical Systems Theory*, 6, 97 – 102, (1972).
- [25]. S. Radenović, F. Vetro and J. Vujaković, “An alternative and easy approach to fixed-point results via simulation functions”, *Demonstration Mathematical*, 50, 223 – 230, 2017.